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PANDIAGONAL PRIME NUMBER MAGIC SQUARES.<sup>1</sup>

For ordinary magic squares the only requirements are that all rows, all columns, and the two main diagonals sum to the constant. For pandiagonal magics it is required in addition that *all* diagonals also sum to the constant.

## PANDIAGONAL MAGIC SQUARES OF EVEN ORDERS.

Dr. C. Planck has given us the following rule for transposing all associated magic squares of even orders into pandiagonal magics but the inverse change from pandiagonal to association does not follow in every case.<sup>2</sup>

A	B
C	D

Fig. 1.

73	41	13	113
23	103	83	31
107	7	47	79
37	89	97	17

Fig. 2.

197	71	83	163	37	79
17	97	109	199	179	29
167	151	53	19	103	137
47	173	131	13	139	127
11	31	181	193	113	101
191	107	73	43	59	157

Fig. 3.

271	1	227	353	101	163	503	421
491	439	151	337	137	61	311	113
269	383	47	197	457	443	11	233
131	23	479	281	467	179	79	401
409	347	7	89	239	509	283	157
373	449	199	397	19	71	359	173
53	67	499	277	241	127	463	313
43	331	431	109	379	487	31	229

Fig. 4.

<sup>1</sup> The author is indebted to Messrs. W. S. Andrews and H. A. Sayles, both of Schenectady, N. Y.: to the former for valuable assistance in writing these papers and to the latter for execution of the diagrams.

<sup>2</sup> See *The Monist* for July, 1910, Vol. XX, No. 3, "The Method of Complementary Differences," by W. S. Andrews.

*Rule:* Divide the square into equal quarters parallel to its sides (see Fig. 1).

Leave A untouched.

Reflect B.

Invert C.

Reflect and invert D.

By this rule the associated magic squares of the fourth, sixth, eighth and tenth orders, (Figs. 2, 3, 4, 5), were transposed from the associated squares shown in the preceding article.

953	607	113	349	919	347	827	337	181	317
127	257	751	421	727	131	23	907	347	659
167	757	571	929	137	547	761	7	103	971
397	109	359	179	739	709	373	977	719	389
557	307	541	911	491	467	53	107	839	677
643	163	653	809	673	37	383	877	641	71
859	967	83	43	331	863	733	239	569	263
443	229	983	887	19	823	233	419	61	853
281	617	13	271	601	593	881	631	811	251
523	937	883	151	313	433	683	449	79	499

Fig. 5.

#### PANDIAGONAL MAGIC SQUARES OF ODD ORDERS.

It is obviously impossible to construct a pandiagonal magic square of the third order since all squares of this order are made from the same formula which is not a pandiagonal one.

<i>x</i>	<i>y</i>	<i>z</i>	<i>v</i>	<i>t</i>
<i>v</i>	<i>t</i>	<i>x</i>	<i>y</i>	<i>z</i>
<i>y</i>	<i>z</i>	<i>v</i>	<i>t</i>	<i>x</i>
<i>t</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>v</i>
<i>z</i>	<i>v</i>	<i>t</i>	<i>x</i>	<i>y</i>

Fig. 6.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
<i>c</i>	<i>d</i>	<i>e</i>	<i>a</i>	<i>b</i>
<i>e</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>a</i>
<i>d</i>	<i>e</i>	<i>a</i>	<i>b</i>	<i>c</i>

Fig. 7.

1	19	53	101	163
71	157	67	13	29
79	23	47	127	61
103	31	73	89	41
83	107	97	7	43

Fig. 8.

Pandiagonal magics of odd prime orders larger than the third can be made from La Hirian primaries constructed as shown in Figs. 6 and 7, the paths in this instance being knight's moves in different directions.

A pandiagonal magic square of the fifth order is shown in Fig. 8 in which

$x=1$   
 $y=13$   
 $z=23$   
 $v=41$   
 $t=97$

$a=0$   
 $b=6$   
 $c=30$   
 $d=60$   
 $e=66$

and the summation = 337.

1	19	53	101	163	751	857
601	653	211	283	23	47	127
311	97	547	617	61	79	233
43	83	107	307	811	587	7
797	271	13	29	71	157	607
103	571	647	67	223	293	41
89	251	367	541	593	31	73

Fig. 9.

Fig. 9 shows a pandiagonal  $7^2$ ,  $S=1945$ , the primaries being as follows:

$x=1$   
 $y=13$   
 $z=23$   
 $v=41$   
 $t=97$   
 $s=541$   
 $p=587$

$a=0$   
 $b=6$   
 $c=30$   
 $d=60$   
 $e=66$   
 $f=210$   
 $g=270$

After long perseverance the writer succeeded in finding the unbalanced series of primes for a magic square of the 9th order,

shown in Fig. 10, but there appears to be no rule for making a pandiagonal magic therefrom. It was therefore decided to approach pandiagonal or nasik results as nearly as possible.

	6	4	136	44	72	82	136	822
12	1	7	11	137	181	251	631	827
18	13	19	23	149	193	263	643	839
30	31	37	41	167	211	281	661	857
42	61	67	71	197	241	311	691	887
330	103	109	113	239	283	353	733	929
1050	433	439	443	569	613	683	1063	1259
858	1483	1489	1493	1619	1663	1733	2113	2309
9306	2341	2347	2351	2477	2521	2591	2971	3167
	5647	5653	5657	5783	5827	5937	6277	6473
								8233

Fig. 10.

826	6	180	2646	10	250	630	0	136
630	0	136	826	6	180	2646	10	250
2646	10	250	630	0	136	826	6	180
6	180	826	10	250	2646	0	136	630
0	136	630	6	180	826	10	250	2646
10	250	2646	0	136	630	6	180	826
180	826	6	250	2646	10	136	630	0
136	630	0	180	826	6	250	2646	10
250	2646	10	136	630	0	180	826	6

Fig. 11.

1	13	31	61	103	433	1483	2341	5647
0	6	10	136	180	250	630	826	2646

The above La Hirian primaries being laid out in the usual way from the unbalanced series, two primary squares (Figs. 11

and 12) were made according to an arrangement formulated by Mr. H. A. Sayles, Schenectady, N. Y. A combination of these two primary squares produced the magic square shown in Fig. 13,

433	31	5647	61	1	1483	103	13	2341
61	1	1483	103	13	2341	433	31	5647
103	13	2341	433	31	5647	61	1	1483
5647	433	31	1483	61	1	2341	103	13
1483	61	1	2341	103	13	5647	433	31
2341	103	13	5647	433	31	1483	61	1
31	5647	433	1	1483	61	13	2341	103
1	1483	61	13	2341	103	31	5647	433
13	2341	103	31	5647	433	1	1483	61

Fig. 12.

$S = 14,979$ . In this square each third diagonal  $= S$ , so that only six diagonals are regular. Each pair of irregular diagonals shows summations  $s + x$  and  $s - x$ ;  $x$  being different with each

1259	37	5827	2707	11	1733	733	13	2477
691	1	1619	929	19	2521	3079	41	5397
2749	23	2591	1063	31	5783	887	7	1663
5653	613	857	1493	311	2647	2341	239	643
1483	197	631	2347	283	839	5657	633	2677
2351	353	2659	5647	569	661	1489	241	827
211	6473	439	251	4129	71	149	2971	103
137	2113	61	193	3167	109	281	8293	443
263	4387	113	167	6277	433	181	2309	67

Fig. 13.

pair in this square. The writer has attempted to correct these faulty diagonals, but so far without success.

In order to build a nasik square of the 9th order by rule, it is

necessary to find a series which will permit the two sets of La Hirian primaries to be each divided into three groups of numbers having a common summation, as exemplified in the arithmetical series 1, 2, 3, 4, . . . . .81. In this series the two sets of La Hirian primaries are

1	2	3	4	5	6	7	8	9
0	9	18	27	36	45	54	63	72

and the above may be rearranged in three groups as follows:

1	5	9		3	4	8		2	6	7
0	36	72		18	27	63		9	45	54

in which each triplet in the upper line sums 15 and each triplet in the lower line sums 108. The difficulty of finding a  $9^2$  series of prime numbers that will meet the above conditions appears to be insurmountable.

The writer believes the squares in this paper to be of the lowest possible summations, but no claim to that effect is made except in the case of the  $4^2$ , though it is probable that  $5^2$  and  $7^2$  are also of minimum summation.

It is hoped that some student of magic squares may be able before long to make a pandiagonal square of the 9th order with prime numbers.

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### PANELED MAGIC SQUARES.\*

These squares are made with a central magic square having one or more panels or borders of figures, so arranged that each enlargement forms another magic square.

Paneled magic squares may be either "perfect" or "imperfect," the former being those in which all intermediate squares are magic, and the latter those wherein one or more of the intermediate squares are not magic.

These two varieties are constructed by different rules. The "perfect" squares are formed entirely of couplets; with the exception of the center cells of odd squares, and the inner  $4^2$  of even squares,

\* Diagrams by Mr. H. A. Sayles, Schenectady, N. Y.